Very high-energy muons in neutrino water (ice) telescopes

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Differential muon energy spectra (for vertical direction) measured in various experiments

At the energies above 10 TeV muon spectrum is poorly studied, the data often contradict each other.
The region of muon energies above 100 TeV is of a special interest.

Firstly, prompt muons from decays of charmed and other short-lived heavy particles produced as a result of the interactions of primary particles with air nuclei, can give appreciable contribution to the muon flux.

Secondly, muon energies of the order of hundreds TeV correspond to PeV energies of primary cosmic ray particles, where changes in the behavior of various EAS components are observed. So, the knee in the energy spectrum of primary cosmic ray particles should lead to a decrease in the muon flux. On the contrary, if the observed changes in the EAS development are connected with inclusion of new physical processes, appearance of an excess of very high energy muons is possible.
Methods of muon spectrum investigations at energies above 10 TeV

- **Magnetic spectrometer technique** \( E_\mu \leq 10 \text{ TeV} \)
  
  (MARS, MUTRON, DEIS, ...)

- **Depth-intensity method** \( E_\mu \leq 100 \text{ TeV} \)
  
  (Baksan, Frejus, KGF, LVD, MACRO)

- **Pair meter technique**
  
  (Baksan, BARS, NUSEX)

- **Calorimetric method** (bremsstrahlung cascades)
  
  (Artyomovsk, Baksan, MSU)

Magnetic spectrometer technique and depth-intensity method have restrictions because of technical and physical reasons.

Pair meter technique and calorimetric method do not have upper energy limit.
The flux of VHE muons is very small, therefore setups of large size (mass) are required.
Different models of muon energy spectrum

1. Usual muon spectrum from $\pi$-, $K$-decays in the atmosphere ($\Delta \gamma = 0$, $R = 0$)

\[
\frac{dN_\mu}{dE_\mu d\Omega} = 0.14E_\mu^{-2.7} \left( \frac{E_\mu}{E_k} \right)^{-\Delta \gamma} \left( \frac{1}{1 + \frac{1.1E_\mu \cos \theta}{115}} + \frac{0.054}{1 + \frac{1.1E_\mu \cos \theta}{850}} + R \right) \quad [\text{cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \text{ GeV}^{-1}]
\]

Possible contribution of any fast (in comparison with $\pi$, $K$-decays) processes to generation of muons often is taken into account by means of introduction of the parameter $R$, which is a ratio of the number of prompt muons to the number of charged pions with the same energy at production.

2. Usual spectrum with the knee at 0.3 PeV in muons and $\Delta \gamma = 0.5$

In contrast with EAS observations, inclusive muon spectrum is sensitive mainly to the energy of primary particle per nucleon, thus indicating the knee position in the spectrum of protons. We assumed that the increase of the power index $\Delta \gamma = 0.5$ in the differential proton spectrum at 3 PeV is followed by the same steepening of the muon spectrum at 10 times lower muon energy ($E_k = 0.3$ PeV).

3. Usual spectrum with addition of prompt muons at the level of $R = 5 \times 10^{-4}$

At that, differential spectra of prompt muons and usual muons (from pion and kaon decays) cross each other at the energy about 300 TeV.

4. Usual spectrum with the knee (model 2) and with prompt muons (model 3)
Calculated differential muon energy spectra (4 models)

- **Models:**
  1. Usual $\mu$ from $\pi$, $K$
  2. Usual $\mu$ + knee ($E_k = 0.3$ PeV, $\Delta\gamma = 0.5$)
  3. Usual $\mu$ + prompt ($R = 5 \times 10^{-4}$)
  4. Usual $\mu$ + knee + prompt

- **Experiments:**
  - Frejus, 1994
  - MACRO, 1995
  - LVD, 1998
  - Artyomovsk, 1988
  - Baksan, 1992
  - MSU, 1994
  - Baikal (limit for $\gamma_{\mu} = 2.7$), 2005
  - Baksan (pair meter), 2009
  - IceCube, 2016

\[ E^3 \frac{dN}{dE}, \text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1} \text{GeV}^2 \]

\[ E_\mu, \text{GeV} \]
Calculated integral muon energy spectra at the Earth's surface and at the depth 2 km water

models:

1 - usual $\mu$ from $\pi, K$

2 - usual $\mu$ + knee ($E_k = 0.3$ PeV, $\Delta\gamma = 0.5$)

3 - usual $\mu$ + prompt ($R = 5 \times 10^{-4}$)

4 - usual $\mu$ + knee + prompt

$N_{\mu} (> E_{\mu})$, muons / (1 km$^2 \times 3$ years)

$E_{\mu}$, GeV
Underground (underwater) muon energy spectrum

Main features of the underground spectrum may be illustrated on the basis of the analytical solution assuming the constancy of coefficients in the muon energy loss relation (and neglecting energy loss fluctuations):

\[-dE_\mu / dX = a + bE_\mu\]

For a power-type differential muon energy spectrum at the surface

\[dN_\mu / dE_\mu = N_0 E_\mu^{-(\gamma+1)}\]

one can easily derive the energy spectrum at the depth \(h\):

\[\frac{dN_\mu(h)}{dE_\mu} = N_0 e^{-\gamma bh} \left[ E_\mu + \frac{a}{b} \left(1 - e^{-bh}\right)\right]^{-(\gamma+1)}\]

Relation between the energy \(E_\mu\) of muons at the surface and their energy \(E_h\) after passing through thickness \(h\) of matter (water, ice, rock) is given by:

\[E_\mu = (E_h + a/b)e^{bh} - a/b\]

Energy loss fluctuations due to rare catastrophic collisions with high relative energy transfers, mostly in the processes of muon bremsstrahlung and nuclear interaction, are important. Influence of fluctuations may be taken into account by the introduction of some effective energy loss coefficients \((b_{\text{eff}}\) and \(b'_{\text{eff}}\)) instead of the value \(b\) determining the average energy loss rate.

\[a = 2.5\text{ MeV cm}^2/\text{g};\]
\[b = 3.5 \times 10^{-6}\text{ cm}^2/\text{g};\]
\[b_{\text{eff}}/b = 0.844,\ b'_{\text{eff}}/b = 0.752\]

(for integral spectrum slope \(\gamma = 2.7\))
Influence of energy loss fluctuations on the underground (underwater) muon energy spectrum

Thin lines show muon spectra for case when fluctuations are taken into account (expected fluxes are a bit higher)

models:
1 - usual $\mu$ from $\pi$, K
2 - usual $\mu$ + knee ($E_k = 0.3$ PeV, $\Delta \gamma = 0.5$)
3 - usual $\mu$ + prompt ($R = 5 \times 10^{-4}$)
4 - usual $\mu$ + knee + prompt

$N_{\mu} (> E_{\mu})$, muons / (1 km$^2 \times$ 3 years)

$E_{\mu}$, GeV

at the depth $h = 2$ km water
Integral spectra of cascades produced by muons underwater for different models of muon energy spectrum

Differential energy spectrum of cascade showers produced via muon interactions in a target with mass $M$ is related to the differential muon spectrum as follows. Here $\sigma (\varepsilon, E)$ is the sum of cross sections of electromagnetic muon interaction processes with nuclei (bremsstrahlung, inelastic muon scattering, electron-positron pair production), $\varepsilon$ is cascade energy, $\Omega$ and $T$ are solid angle and observation time:

$$\frac{dN_c}{d\varepsilon} = MT \int_{\varepsilon}^{\infty} \sigma(\varepsilon, E) \frac{dN_\mu(h)}{dEd\Omega} dEd\Omega$$

The expected number of the cascades with energies $> 100$ TeV is of the order of $10^4$.  

Differential number of cascade showers produces by muons underwater for different models of muon energy spectrum.

models:
1 - usual $\mu$ from $\pi, K$
2 - usual $\mu +$ knee ($E_k = 0.3$ PeV, $\Delta\gamma = 0.5$)
3 - usual $\mu +$ prompt ($R = 5*10^{-4}$)
4 - usual $\mu +$ knee + prompt

at the depth $h = 2$ km water
Conclusions

Analytical calculations give possibility to obtain estimates of the spectrum of cascades generated by muons in giant underwater setups quite simply.

The results of calculations showed that such detectors allow to measure the muon energy spectrum in PeV energy range, where the changes of the muon spectrum connected with prompt muon production and the knee in the primary cosmic ray spectrum should be observed.

Another possibility of high energy spectrum measurements may be connected with the use of the pair meter technique – estimation of muon energies via multiple successive pair production in a thick layer of matter.
Thank you for your attention!
Distribution of energy transfer for 100 TeV muon interactions in 1 km of the water

\[ E_\mu = 100 \text{ TeV} \quad N_\mu = 10^6 \]

\[ H_2O \quad X = 1 \text{ km} \]

- bremsstrahlung
- \( \delta \)-electrons
- \( e^+e^- \) pairs
- photonuclear