On the reduction of drift coefficients in the presence of turbulence

N.E. Engelbrecht\textsuperscript{1}, R.D. Strauss\textsuperscript{1}, J.A. le Roux\textsuperscript{2}, and R.A. Burger\textsuperscript{1}

\textsuperscript{1}Center for Space Research, North-West University, Potchefstroom, South Africa
\textsuperscript{2}Center for Space Plasma and Aeronomics Research, University of Alabama in Huntsville, Huntsville, AL 3585, USA

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Drift effects have long been known to play a significant role in cosmic ray modulation (22-year cycle, etc.).

The drift coefficient, which enters the TPE via the off-diagonal elements of the diffusion tensor, can, in the weak scattering limit, be expressed by e.g. Forman et al. (1974) as

$$\kappa_A^{ws} = \frac{v}{3} R_L,$$

with $R_L$ the maximal gyroradius and $v$ the particle speed.

Cosmic ray modulation studies have long employed an ad hoc form for the reduced drift coefficient, given by

$$\kappa_A = \frac{\beta P}{3B_0} \frac{(P/P_0)^2}{1 + (P/P_0)^2},$$

due to the fact that the weak scattering result above overestimates drift effects.
Turbulence reduces drift

Numerical test particle simulations, where the Newton-Lorentz equation is solved for an ensemble of test particles in various pre-specified turbulent magnetic field conditions, do reveal some details as to the exact nature of the reduction of the drift coefficient. We focus on two studies here....

Minnie et al. (2007) studied this effect for both a uniform background magnetic field as well as a background field with an imposed spatial gradient, finding the same levels of reduction for the drift coefficient in each case when the same turbulence conditions are used. They also report a reduction in the drift velocity of particles.

Tautz & Shalchi (2012) performed simulations of the drift coefficient for different turbulent geometries and different wavenumber-dependencies of the energy-containing range on the turbulence power spectrum. For isotropic and composite turbulence the drift coefficient is essentially the weak scattering coefficient for very low levels of turbulence, becoming more reduced as turbulence levels increase. Pure slab turbulence does not reduce the computed drift coefficient from the weak scattering value.
Bieber & Matthaeus (1997) (BAM97) find that

\[ \kappa_A = \frac{v}{3} R_L \frac{\Omega^2 \tau^2}{1 + \Omega^2 \tau^2}, \text{ where } \Omega \tau = \frac{2}{3} \frac{R_L}{D_\perp}. \]

The perp. FLRW diffusion coefficient is

\[ D_\perp = \frac{1}{2} \left( D_{sl} + \sqrt{D_{sl}^2 + 4D_{2D}^2} \right), \text{ where} \]

\[ D_{sl} = \frac{1}{2} \frac{\delta B_s}{B_o^2} \frac{\lambda_{c,s}}{\lambda_{u,2D}} \text{ and } D_{2D} = \frac{\sqrt{\delta B_{2D}^2}}{B_o} \lambda_{u,2D}. \]

Burger & Visser (2010) (BV2010) propose a parametrized form:

\[ \Omega \tau = \frac{11}{3} \frac{R_L}{\lambda_{c,s}} D_\perp \left( \lambda_{c,s} / \lambda_{c,s} \right)^g, \]

with

\[ g = 0.3 \log \left[ \frac{R_L}{\lambda_{c,s}} \right] + 1.0. \]

The BV2010 result clearly fits the Minnie et al. simulations better than the BAM97 coefficient... but it is moot whether it would be applicable for different turbulence conditions, etc.
**Turbulence reduces drift**

- Tautz & Shalchi (2012) find that, for pure slab turbulence, the drift coefficient remains unreduced, regardless the level of turbulence assumed.

- These authors also report a relatively weak dependence of the drift-reduction factor on particle rigidity and on the energy-range spectral index of the 2D fluctuation spectrum.

- Furthermore, they propose a parametrized form:

\[
\kappa_A = \frac{\nu}{3} R_L \frac{1}{1 + a (\delta B_T^2 / B_0^2)^d},
\]

with \(a\) and \(d\) fitting constants that change with different turbulence geometries assumed in the simulations, and \(\delta B_T^2\) the (total) magnetic variance.

- Such a fit, though relatively tractable, suffers from the same limitations as that proposed by Burger & Visser (2010).
The approach taken by Bieber & Matthaeus (1997)

BAM97 use the standard TGK approach to calculate diffusion coefficients from an ensemble of particle trajectories:

\[ D_{ij} = \int_0^\infty dt R_{ij}(t) \]

where \( R_{ij}(t) = \langle v_i(t_o) v_j(t_o + t) \rangle \) is the velocity correlation function, assumed to be independent of the reference time \( t_o \), and to go to zero at a rate greater than \( 1/t \) as \( t \) goes to infinity. Then \( D_{xy} = -D_{yx} = \kappa A \) for diffusive particle behaviour.

Exact calculation of the above is difficult. BAM97 consider first the correlation function when no turbulence is present. Then \( R_{yx} = -R_{xy} \sim \sin \Omega t \).

These authors assume that turbulence causes a particle to 'forget' its original trajectory, so \( R_{ij} \) should drop to zero after a sufficient time. This is modelled using a decorrelation rate \( f_\perp = \tau^{-1} \) so that

\[ R_{yx} = \frac{v^2}{3} \sin(\Omega t) e^{-f_\perp t} \]

with \( \Omega \) the gyrofrequency of the unperturbed particle and \( v \) its speed.

Integration then yields

\[ \kappa A = \frac{v R_L}{3} \frac{(\Omega \tau)^2}{1 + (\Omega \tau)^2} \]

The problem now is to suitably model the decorrelation time \( \tau \).
BAM97 argue that the field line random walk process will be the major factor in the perpendicular decorrelation process, introducing a lengthscale \( z_c = \frac{R_L^2}{D_\perp} \) over which the perpendicular correlation function would significantly decrease.

This then leads to a decorrelation time of

\[
\tau \sim \frac{R_L^2}{vD_\perp}.
\]

Here, we do not assume that decorrelation is entirely due to FLRW, as the drift process would act so as to cause particles to leave field lines. We assume that the perpendicular decorrelation scale is inversely proportional to some lengthscale along which decorrelation perpendicular to the uniform background field occurs, which we approximate as the particle’s perpendicular mean free path, so that \( z_c = \frac{R_L^2}{\lambda_\perp} \). The choice of \( \lambda_\perp \), as opposed to the turbulence correlation length, is motivated by the fact that we are interested in the particle velocity decorrelation in particular.

We assume that the perpendicular decorrelation rate is influenced only by the particle’s speed perpendicular to the uniform background field \( v_\perp \). This then gives the decorrelation time as

\[
\tau = \frac{R_L^2}{v_\perp \lambda_\perp}.
\]
An alternative....

To estimate $v_\perp$, consider a Reynold’s decomposed turbulent magnetic field in two dimensions $\vec{B} = B_0 \vec{e}_z + b_x \vec{e}_x$, where $B_0$ is uniform, $b_x$ a fluctuating, transverse component, and $\langle B \rangle = B_0$.

At any particular point along $\vec{B}$, if $\theta$ is the angle between $\vec{B}$ and $B_0 \vec{e}_z$, we have that $\sin \theta = b_x / B \approx b_x / B_0$, assuming small fluctuations.

This angle will be the same as the average angle between the particle velocity $\bar{v}$ and its component parallel to $\vec{e}_z$, such that $\sin \theta = v_x / v$, again assuming small fluctuations.

This then leads to $v_x \approx v (b_x / B_0)$. As it follows that $\langle v_x \rangle = 0$, we then model $v_\perp$ as the root-mean-square value of this quantity. Therefore, we use $v_\perp \approx v (\delta B_T / B_0)$.

Then we have that

$$\Omega \tau = \frac{R_L}{\lambda_\perp} \frac{B_0}{\delta B_T}.$$  

This then then yields

$$\kappa_A = \frac{v R_L}{3} \left[ 1 + \frac{\lambda_\perp^2 \delta B_T^2}{R_L^2 B_0^2} \right]^{-1}.$$
Comparison with simulation data: $f_s$ as function of turbulence level

![Graph showing $f_s$ as function of $(\delta B/B_0)^2$ with data points and curves for different $R/L_\lambda_c$ ratios.](image-url)
Comparison with simulation data: $v_d$ as function of turbulence level
Large-scale MHD outputs from Wiengarten et al. (2016)
Turbulence model results from Wiengarten et al. (2016)
The perpendicular mean free path, based on Ruffolo et al. (2012)

\[ \lambda_\perp = \frac{C_0 \lambda_{2D} \delta B_{2D}^2}{B \epsilon \lambda_{\parallel}} \left[ h_{\perp,1} + h_{\perp,2} + h_{\perp,3} \right], \]

where

\[ h_{\perp,1} = \frac{1}{q} \left[ a \sqrt{3 \pi \epsilon \lambda_{\text{par}}} \text{erfc} (x_1) - 3B \lambda_{\text{out}} E_{(q+1)/2} (x_1^2) \right], \]

\[ h_{\perp,2} = 6B \left( \lambda_{2D} x_2 - \lambda_{\text{out}} x_3 \right) + a \sqrt{3 \pi \epsilon \lambda_{\text{par}}} \log \left( \frac{\lambda_{\text{out}}}{\lambda_{2D}} \right), \]

\[ h_{\perp,3} = \frac{a \sqrt{3 \pi \epsilon \lambda_{\text{par}}}}{\nu} \left[ \frac{x_4^{-1}}{\sqrt{\pi}} \left( \Gamma \left( \frac{\nu + 1}{2} \right) - \Gamma \left( \frac{\nu + 1}{2}, x_4^2 \right) \right) + \text{erfc} (x_4) \right], \]

with, for notational convenience

\[ x_1 = \frac{\sqrt{3}B \lambda_{\text{out}}}{a \sqrt{\epsilon \lambda_{\text{par}}}}, \]

\[ x_2 = 2F_2 \left( \frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -x_4^2 \right), \]

\[ x_3 = 2F_2 \left( \frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -x_1^2 \right), \]

\[ x_4 = \frac{\sqrt{3}B \lambda_{2D}}{a \sqrt{\epsilon \lambda_{\text{par}}}}. \]
Mean free paths, drift scales at Earth as function of rigidity

![Graph showing mean free paths, drift scales at Earth as function of rigidity. The x-axis represents rigidity (P [GV]) and the y-axis represents mean free path (λ [AU]). The graph includes several lines representing different models and conditions, such as BV2010, TS2012, New, Parallel MFP, Perpendicular MFP, and Larmor radius. The graph covers a range of rigidity from 0.1 to 100 GV and mean free path from 10^-6 to 1 AU.]
Mean free paths, drift scales at 90AU as function of rigidity

- BV2010
- TS2012
- New
- Parallel MFP
- Perpendicular MFP
- Larmor radius

Graph showing the variation of mean free paths (\(\lambda\)) in AU as a function of rigidity (\(P\)) in GV for different models and parameters.
Drift scales at 0.01 GV - Comparisons

**New**

**Weak**
To conclude....

- We now have a relatively simple, tractable way to model the effects of turbulence on cosmic ray drift coefficients.

- The new drift coefficient compares reasonably well with existing numerical simulations of this quantity.

- In theory, this drift coefficient can be applied in turbulence scenarios different to those at 1 AU, but more simulations need to be done to test this.

- Using the latest turbulence transport model to provide inputs for various turbulence quantities throughout the heliosphere, the new coefficient yields results that differ significantly from those of previously proposed coefficients. Given the sensitivity of computed CR intensities to the choice of drift coefficient (see Engelbrecht & Burger, 2015, AdSpR, 55, 390), this new coefficient would be of great interest to CR modulation studies.

- For an example of an implementation of this new drift coefficient in a CR modulation study, see the talk by K.D. Moloto on Monday the 17th.